



# 一维热传导-对流方程的解析解

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# 耦合热传导对流方程

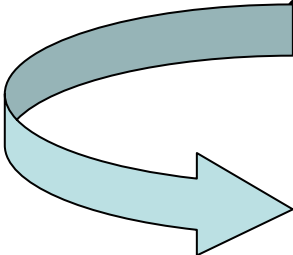
$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} + W \frac{\partial T}{\partial z}$$

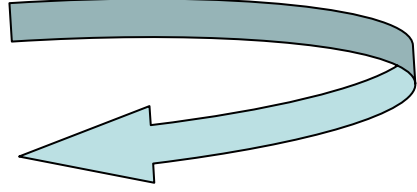
热传导项

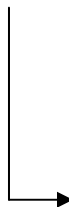
热对流项

$$W = -\frac{C_w}{C_g} w \theta$$

Gao. Z., X. Fan, and L. Bian. 2003:  
An Analytical Solution to One-Dimensional Thermal  
Conduction-Convection in Soil. *Soil Science*, 168(2): 99-107.

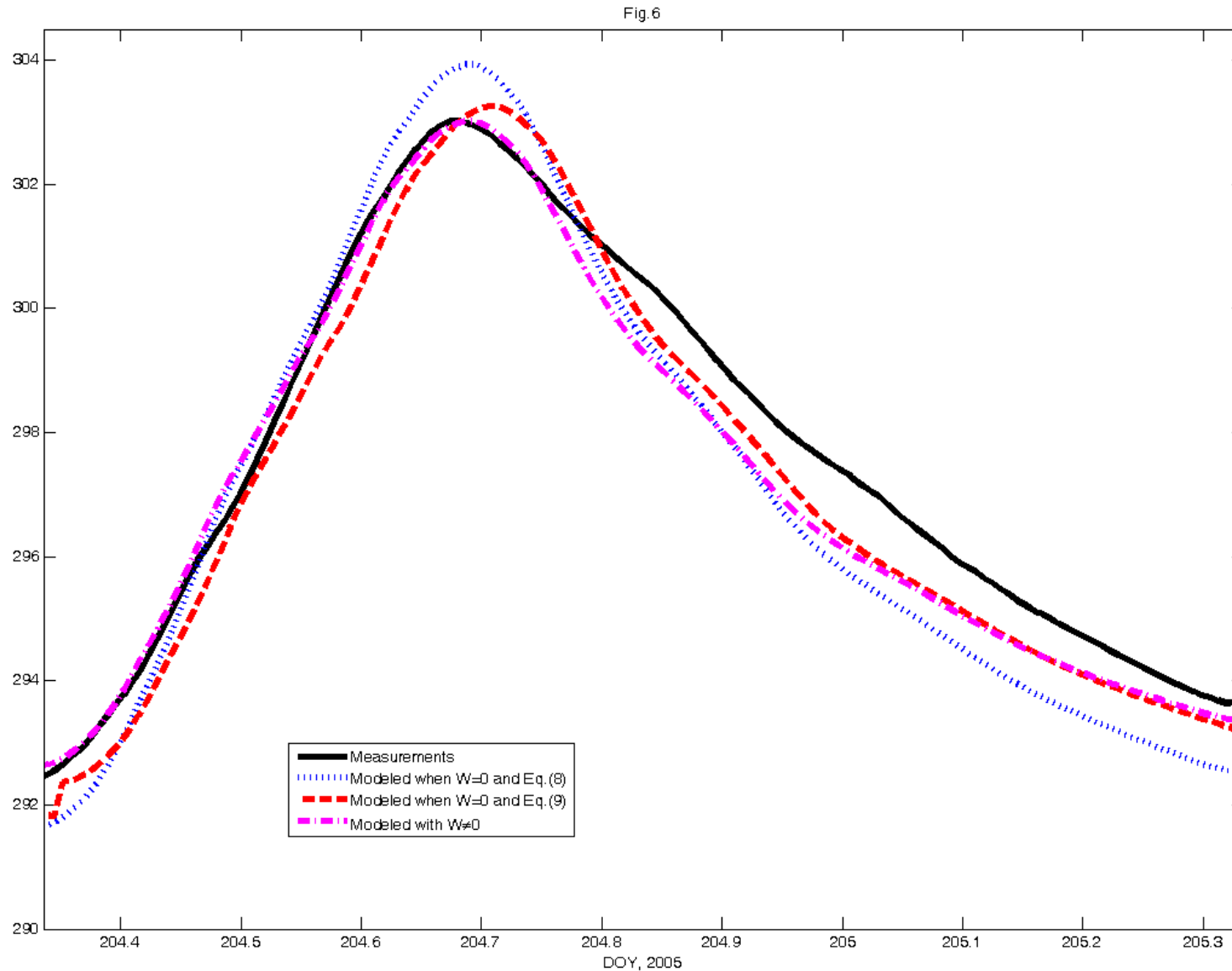
$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_g} \frac{\partial^2 T}{\partial z^2} + \frac{1}{C_g} \frac{\partial \lambda}{\partial z} \frac{\partial T}{\partial z} \approx k \frac{\partial^2 T}{\partial z^2} + \frac{\partial k}{\partial z} \frac{\partial T}{\partial z}$$


$$\frac{\partial T}{\partial t} = -\frac{C_w}{C_g} w \theta \frac{\partial T}{\partial z}$$


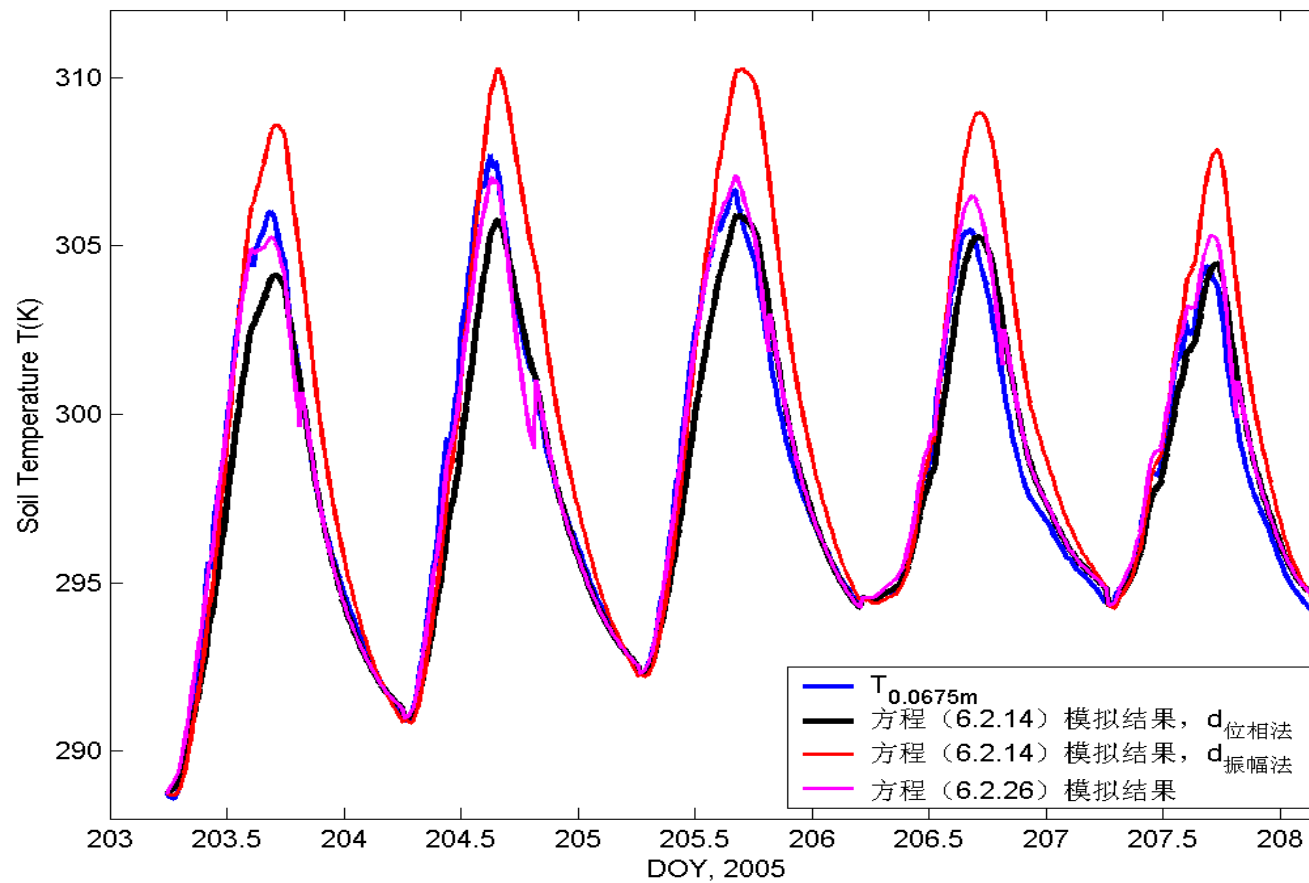
$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} + W \frac{\partial T}{\partial z}$$


$$W \equiv \frac{\partial k}{\partial z} - \frac{C_w}{C_g} w \theta$$

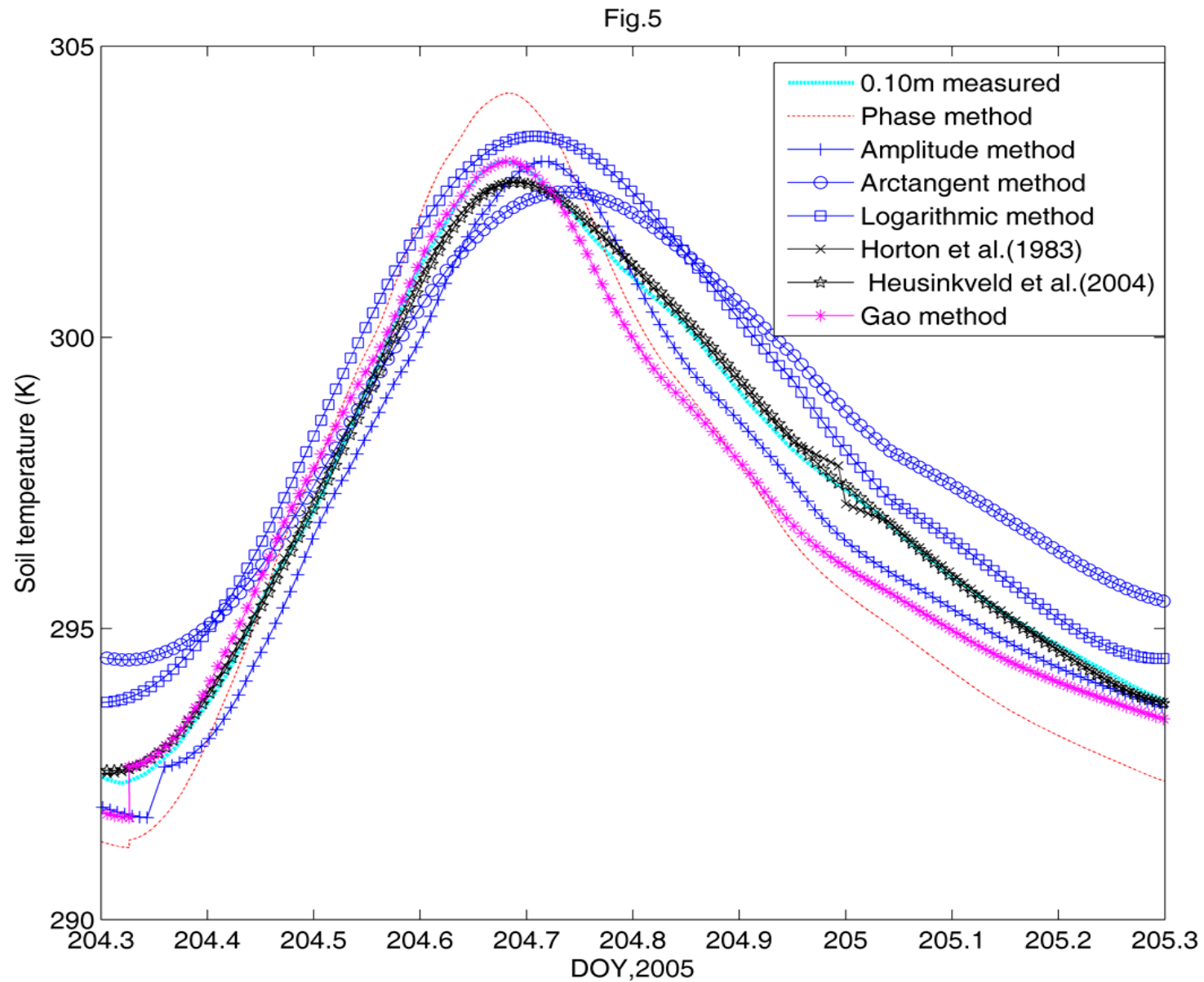
Gao, Z., D. H. Lenschow, R. Horton, M. Zhou, L. Wang, and J. Wen 2008, Comparison of Two Soil Temperature Algorithms for a Bare Ground Site on the Loess Plateau in China, *J. Geophys. Res.*, doi:10.1029/2008JD010285.



Gao, Z. (高志球), D. H. Lenschow, R. Horton, M. Zhou, L. Wang, and J. Wen 2008, Comparison of Two Soil Temperature Algorithms for a Bare Ground Site on the Loess Plateau in China, *J. Geophys. Res.*, doi:10.1029/2008JD010285.

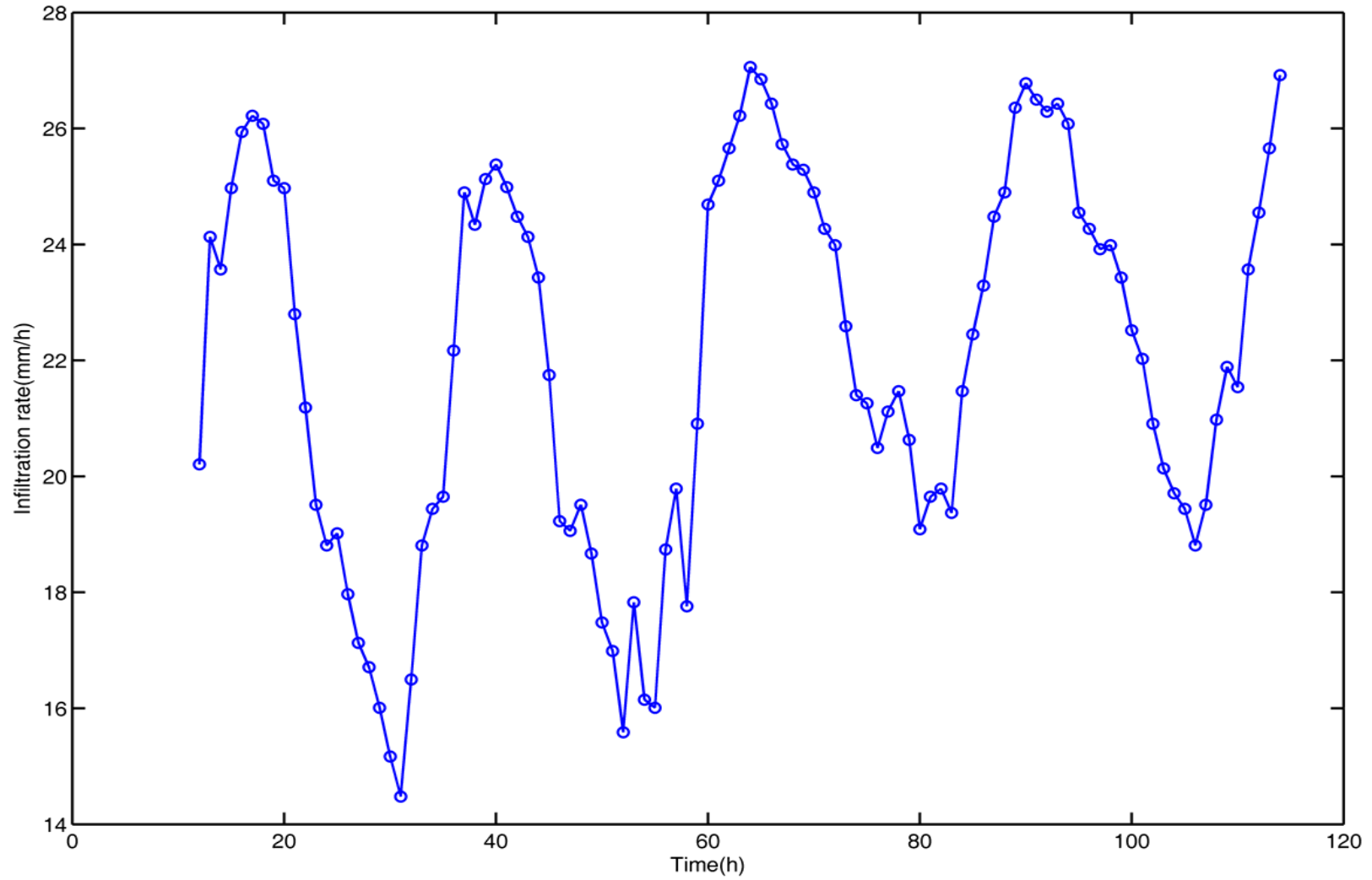


Gao Z., R. Horton, L. Wang, J. Wen, 2008: An Extension of the Force-Restore Method for Soil Temperature Prediction. *European Journal of Soil Science*, doi: 10.1111/j.1365-2389.2008.01060.x



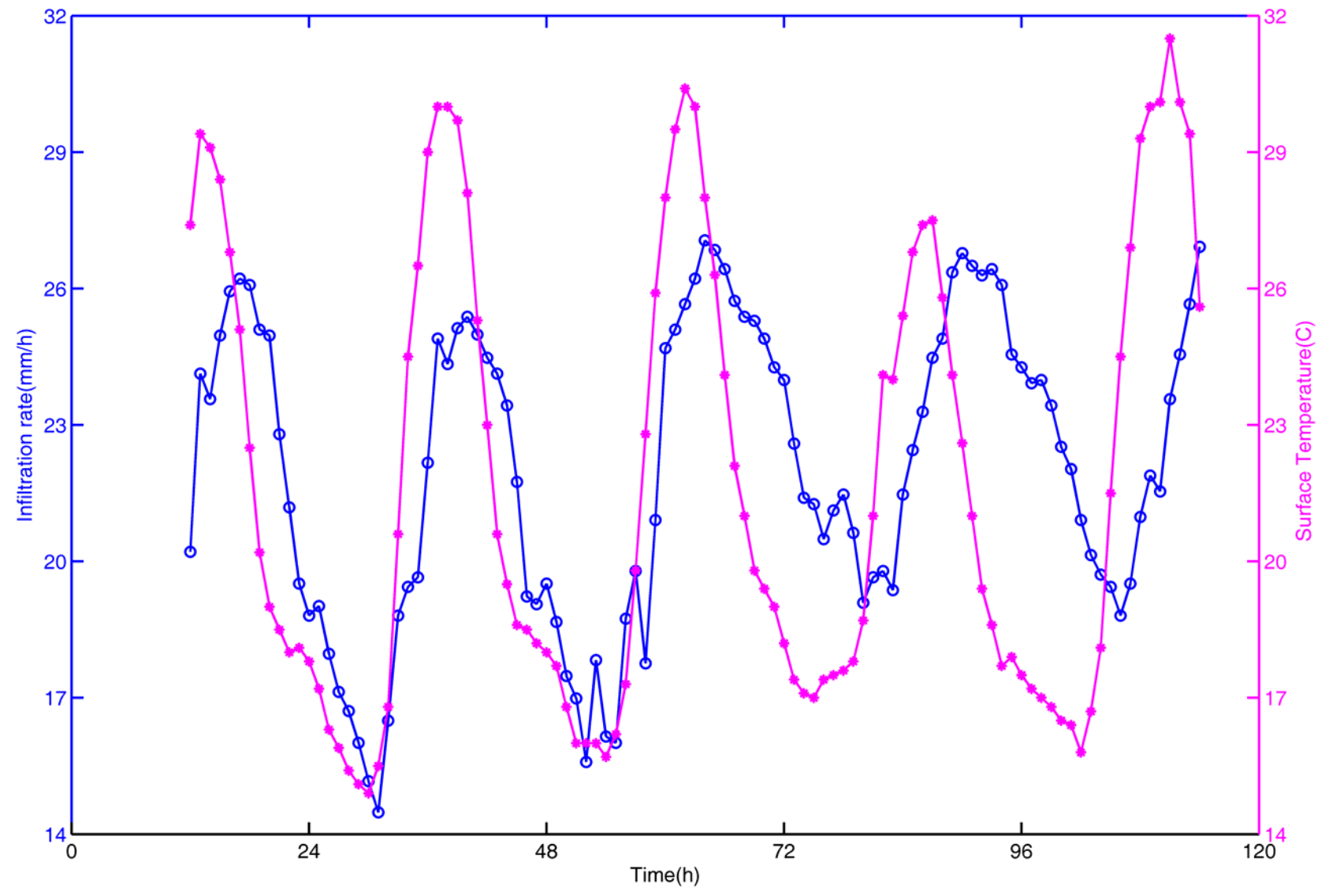
Wang, L., Z. Gao, R. Horton, Comparison of Six Methods to Determine the Surface Soil Thermal Diffusivity by using the data collected at the Bare Ground soil over the Loess Plateau in China.

# 土壤水通量密度



数据来源: Jaynes D. B., 1990: Temperature Variations Effect on Field-Measured Infiltration. *Soil Sci. Soc. Am. J.* 54: 305-312.

# 土壤水通量密度与土壤温度



Buckingham-Darcy 通量定律:

$$W = -K(h) \left( \frac{\partial h}{\partial z} - 1 \right)$$

考虑温度水分通量密度计算公式:

$$W(t) = -\frac{\eta_r}{\eta_t} K_r(h) \left( \frac{\partial h}{\partial z} - 1 \right)$$

$$K_r(h) = K_{sr} \left| \frac{h_a}{h} \right|^n$$

当水流以常量下渗时,  $\partial h / \partial z = 0$  此时水流只在重力梯度下移动

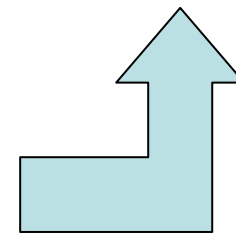
$$K_r(h) = K_{sr}$$

$$W(t) = -\frac{\eta_r}{\eta_t} K_{sr}$$

$$\frac{\eta_r}{\eta_t} = u_0 + u_1 T$$

$$W(t) = a + b \sin \omega t$$

$$T = T_0 + A \sin(\omega t)$$

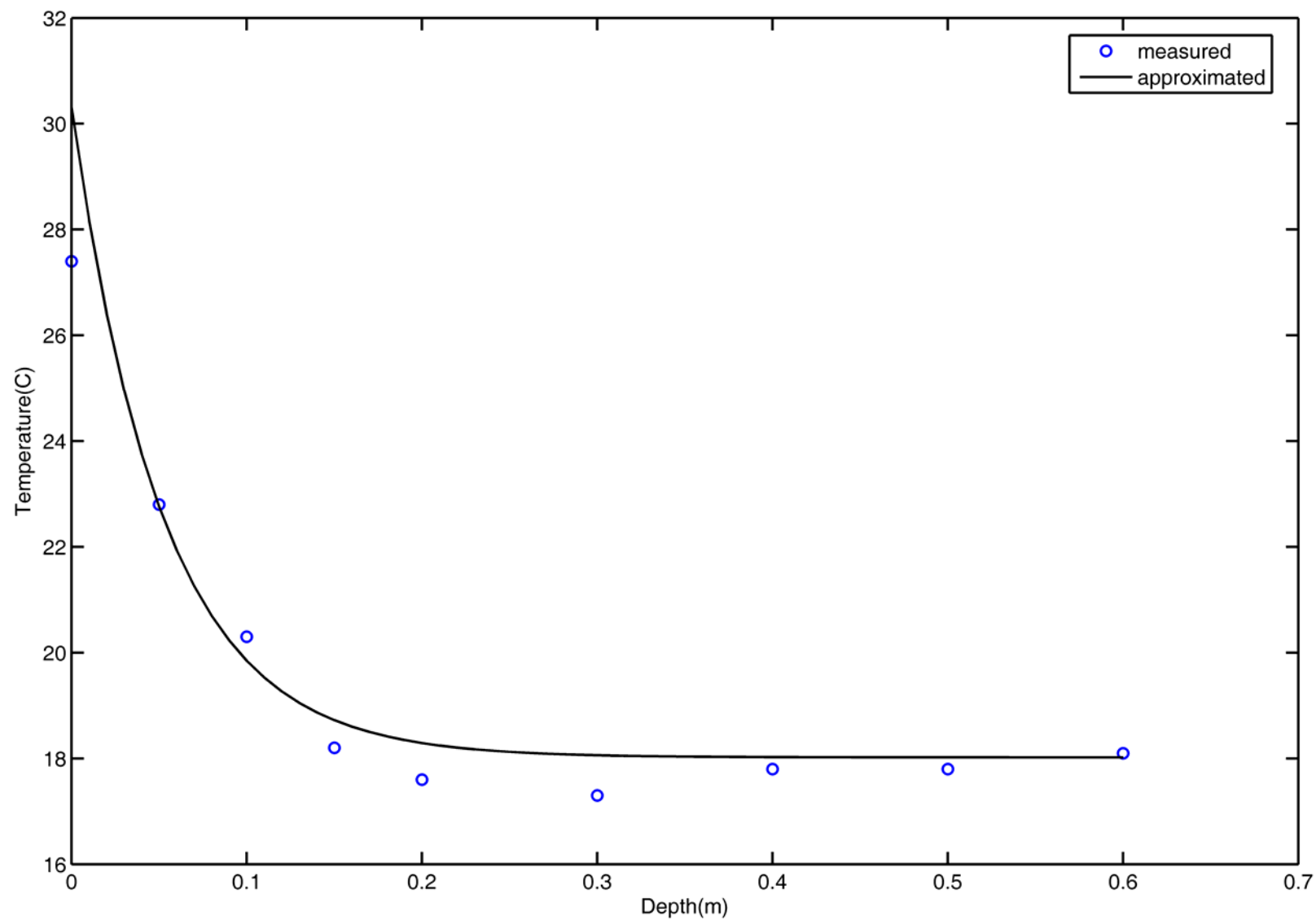


考虑土壤水分垂直运动日变化的土壤温度方程:

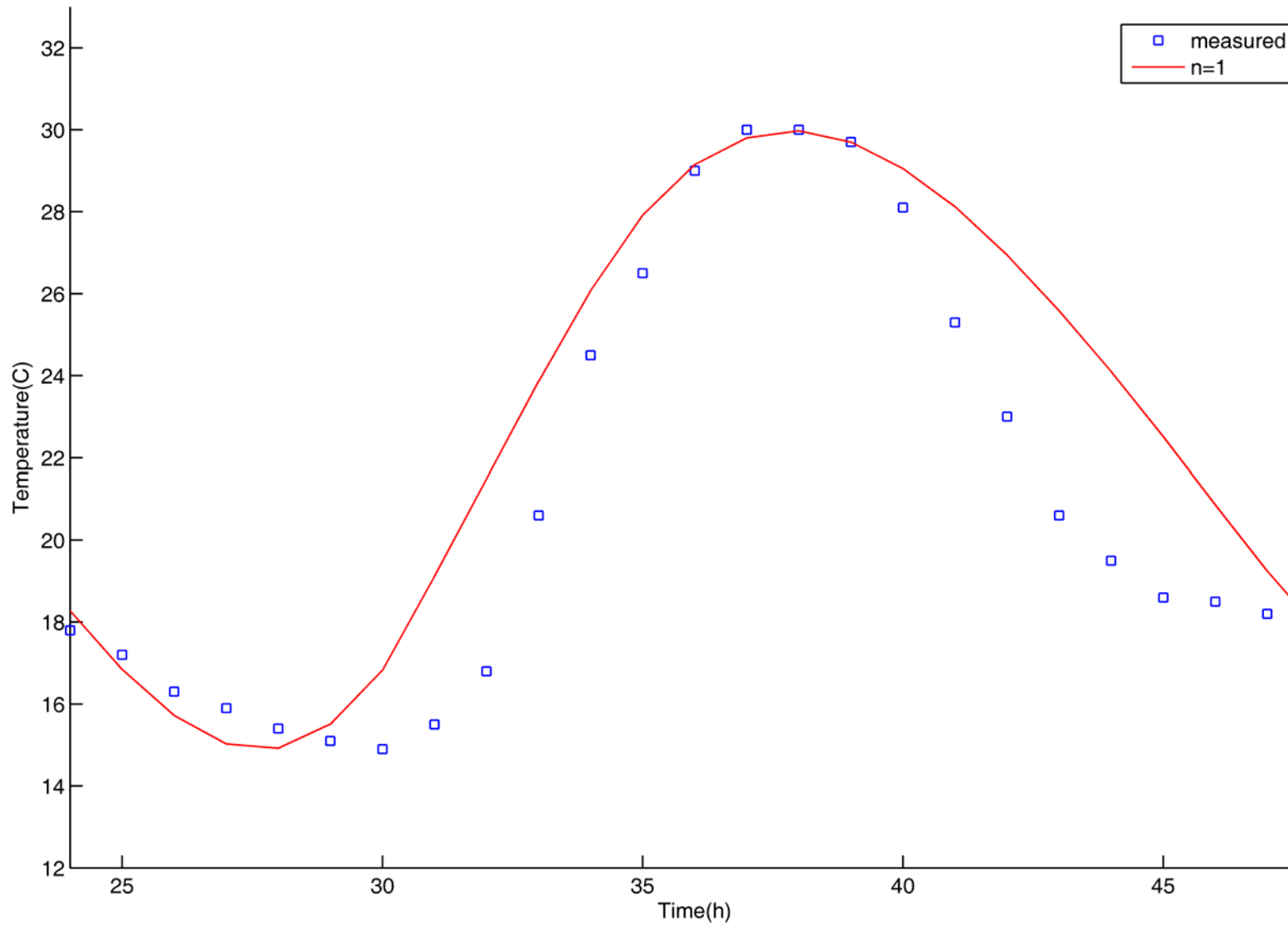
$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - (a + b \sin \omega t) \frac{\partial T}{\partial z}$$

初始条件:

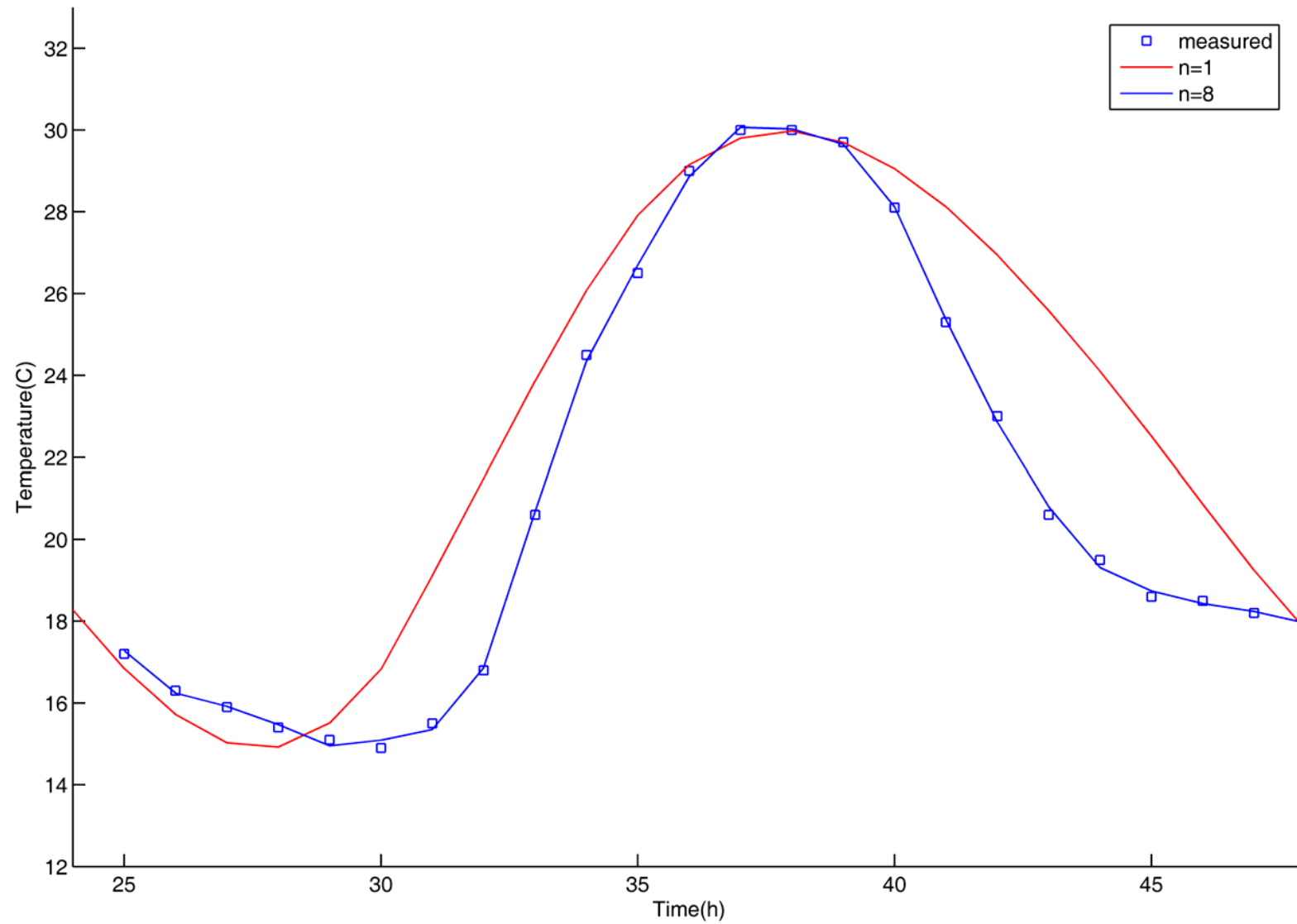
$$f(z) = T_1 + Be^{-kz}$$



边界条件:  $T(0,t) = T_0 + A\sin(\omega t + \Phi)$



边界条件:  $T(0,t) = T_0 + \sum_{i=1}^n A_i \sin(i\omega t + \Phi_i) \quad (n = 1, 2, 3, \dots, n)$



$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - (a + b \sin \omega t) \frac{\partial T}{\partial z}$$

$$T(z, 0) = f(z)$$

$$T(\infty, t) = T_1$$

$$T(0, t) = T_0 + \sum_{i=1}^n A_i \sin(i\omega t + \Phi_i) \quad (n = 1, 2, 3, \dots, n)$$

$$T(z,t) = T_1 + \exp\left(\frac{a_1 z}{2k} - \frac{a_1^2 t}{4k}\right) \frac{2}{\pi} \int_0^\infty V(\lambda,t) \sin(\lambda(x - \lambda_1(t))) d\lambda$$

$$V(\lambda, t) = \exp(-k p^2 t + b_3 \cos \omega t)(V_1 + V_2 + V_3 + V_4 + C)$$

$$V_1 = b_1 \exp(b_2 t) \sum_{i=1}^n \frac{b_2 A_i \sin(i\omega t + \Phi_i) - i\omega A_i \cos(i\omega t + \Phi_i)}{b_2^2 + (i\omega)^2}$$

$$V_2 = -b_1 b_3 \sum_{i=1}^n [\cos \Phi_i V_2'(i) + \sin \Phi_i V_2''(i)]$$

$$V_2'(i) = \frac{1}{2} \frac{\exp(b_2 t)}{b_2^2 + (i+1)^2 \omega^2} [b_2 A_i \sin[(i+1)\omega t] - (i+1)\omega A_i \cos[(i+1)\omega t]]$$

$$+ \frac{1}{2} \frac{\exp(b_2 t)}{b_2^2 + (i-1)^2 \omega^2} [b_2 A_i \cos[(i-1)\omega t] - (i-1)\omega A_i \sin[(i-1)\omega t]]$$

$$V_3 = \frac{b_4}{b_2} \exp(b_2 t)$$

$$V_4 = -b_4 b_3 \exp(b_2 t) \frac{b_2 \cos(\omega t) + \omega \sin(\omega t)}{b_2^2 + \omega^2}$$

$$C = \exp\left(-\frac{a_1 b}{2k\omega}\right) V(\lambda, 0) - [V_1(\lambda, 0) + V_2(\lambda, 0) + V_3(\lambda, 0) + V_4(\lambda, 0)]$$



谢谢!

